nag_bivariate_normal_dist (g01hac)

1. Purpose

nag_bivariate_normal_dist (g01hac) returns the lower tail probability for the bivariate Normal distribution.

2. Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_bivariate_normal_dist(double x, double y, double rho, NagError *fail)
```

3. Description

For the two random variables (X, Y) following a bivariate Normal distribution with

$$E[X] = 0,\ E[Y] = 0,\ E[X^2] = 1,\ E[Y^2] = 1 \text{ and } E[XY] = \rho,$$

the lower tail probability is defined by

$$P(X \leq x, Y \leq y : \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{y} \int_{-\infty}^{x} \exp\left(-\frac{(X^2 - 2\rho XY + Y^2)}{2(1-\rho^2)}\right) dX\, dY.$$

For a more detailed description of the bivariate Normal distribution and its properties see Abramowitz and Stegun (1965) and Kendall and Stuart (1969). The method used is described by Divgi (1979).

4. Parameters

x

Input: the first argument for which the bivariate Normal distribution function is to be evaluated, x.

y

Input: the second argument for which the bivariate Normal distribution function is to be evaluated, y.

rho

Input: the correlation coefficent, ρ.
Constraint: $-1.0 \leq$ **rho** ≤ 1.0.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

On any of the error conditions listed below nag_bivariate_normal_dist returns 0.0.

NE_REAL_ARG_LT

On entry, **rho** must not be less than -1.0: **rho** = ⟨*value*⟩.

NE_REAL_ARG_GT

On entry, **rho** must not be greater than 1.0: **rho** = ⟨*value*⟩.

6. Further Comments

The probabilities for the univariate normal distribution can be computed using nag_cumul_normal (s15abc) and nag_cumul_normal_complem (s15acc).

6.1. Accuracy

Accuracy is discussed in Divgi (1979). A higher order polynomial approximation to Mills ratio is used in nag_bivariate_normal_dist, (15 terms) than is given in Divgi (1979). This will give a higher absolute accuracy of about 10 digits on machines of sufficiently high precision.

6.2. References

Abramowitz M and Stegun I A (1965) *Handbook of Mathematical Functions* Dover Publications, New York ch 26.

Divgi D R (1979) Calculation of univariate and bivariate normal probability functions *Ann. Statist.* **7** (4) 903–910.

Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Vol 1)* Griffin.

7. See Also

nag_cumul_normal (s15abc)
nag_cumul_normal_complem (s15acc)

8. Example

Values of x and y for a bivariate Normal distribution are read along with the value of ρ. The lower tail probabilities are computed.

8.1. Program Text

```
/* nag_bivariate_normal_dist(g01hac) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
  double prob, rho, x, y;

  /*  Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("g01hac Example Program Results\n");
  Vprintf("    x       y      rho    prob\n\n");
  while (scanf("%lf %lf %lf", &x, &y, &rho) != EOF)
    {
      prob = g01hac(x, y, rho, NAGERR_DEFAULT);
      Vprintf("%8.3f%8.3f%8.3f%8.4f\n", x, y, rho, prob);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01hac Example Program Data
  1.7  23.1   0.0
  0.0   0.0   0.1
  3.3  11.1   0.54
  9.1   9.1   0.17
```

8.3. Program Results

```
g01hac Example Program Results
    x        y       rho    prob

   1.700  23.100   0.000  0.9554
   0.000   0.000   0.100  0.2659
   3.300  11.100   0.540  0.9995
   9.100   9.100   0.170  1.0000
```